Math 242
Spring 2019
Hangelbroek

Math 242, Midterm 2

Name:____ Circle section: **5** **6**

Question	Points	Score
1	20	
2	10	
3	10	
4	16	
5	30	
6	14	
Total:	100	

INSTRUCTIONS: Put away all electronics. Check that your exam has all eight pages. Write your name on your exam. Please read all directions carefully and thoroughly. Showing more work will make you more likely to earn partial credit in the event that you made an error.

Good luck! :)

1. Evaluate the integrals. Be sure to indicate if the integral is improper.

   (a) (10 points) $\displaystyle\int_4^8 \frac{x\,\mathrm{d}x}{x^2 - 2x - 3}$

(b) (10 points) $\displaystyle\int_{-2}^{14} \frac{1}{\sqrt[4]{x+2}}\mathrm{d}x$

2. (10 points) The error for the n-term trapezoid rule approximation to $\int_a^b f(x)dx$ can be estimated by $E_n \leq \frac{(b-a)^3 M}{12n^2}$, where M is the maximum value of $|f''(x)|$ over the interval $[a, b]$. Suppose you want to compute the integral

$$\int_{-\sqrt[3]{6}}^{\sqrt[3]{6}} \cos(x + \pi)\, dx$$

with tolerance 0.01. How big should you choose n?

3. Find the limits of each sequence:
   (a) (5 points) $a_n = \frac{\sin(3n)}{3n-1}$.

   (b) (5 points) $a_n = \sqrt[n]{2n+1}$.

4. Evaluate each series:
   (a) (8 points) $\displaystyle\sum_{n=3}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$.

   (b) (8 points) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{3^n}$

5. For the following problems, determine if the series is convergent or divergent. Carefully justify each answer.

(a) (10 points) $\displaystyle\sum_{n=4}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

(b) (10 points) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n+2\sqrt{n}}$

(c) (10 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$

6. Consider the alternating series

$$\sum_{n=1}^{\infty}(-1)^n \frac{1+n}{n^2}$$

(a) (5 points) Does the series converge?

(b) (5 points) Does the series converge absolutely?

(c) (4 points) Estimate the error between $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{1+n}{n^2}$ and the partial sum $s_4 = \displaystyle\sum_{n=1}^{4}(-1)^n \frac{1+n}{n^2}$.

Formula sheet

- Derivatives of inverse trigonometric functions.

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{x\sqrt{x^2-1}}$$

- Trigonometric identities.

$$\begin{aligned}
\sin^2 x + \cos^2 x &= 1 \\
1 + \tan^2 x &= \sec^2 x \\
1 + \cot^2 x &= \csc^2 x \\
\sin^2 x &= \frac{1}{2}(1-\cos(2x)) \\
\cos^2 x &= \frac{1}{2}(1+\cos(2x)) \\
\sin x \cos x &= \frac{1}{2}\sin(2x) \\
\sin x \sin y &= \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y) \\
\cos x \cos y &= \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y) \\
\sin x \cos y &= \frac{1}{2}\sin(x-y) + \frac{1}{2}\sin(x+y)
\end{aligned}$$

- Integrals of trigonometric functions.

$$\begin{aligned}
\int \tan x \; dx &= \ln|\sec x| + C \\
\int \cot x \; dx &= \ln|\sin x| + C \\
\int \sec x \; dx &= \ln|\sec x + \tan x| + C \\
\int \csc x \; dx &= -\ln|\csc x + \cot x| + C
\end{aligned}$$

- Error estimates for Trapezoidal Rule and Simpson's Rule.

$$|E_T| \le \frac{M(b-a)^3}{12n^2}, \quad \text{where } |f''(x)| \le M \text{ for all } x \text{ in } [a,b]$$

$$|E_S| \le \frac{M(b-a)^5}{180n^4}, \quad \text{where } |f^{(4)}(x)| \le M \text{ for all } x \text{ in } [a,b]$$